# Prediction of Exotic Islands of Deformation in the Generalized Differential Equation Model 

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#### Abstract

Predictions for possible occurrence of exotic islands of deformation in the neutronand proton-rich regions of the nuclear chart are made from the calculated values of the reduced quadrupole transition probability $\mathrm{B}(E 2) \uparrow$ for the transition from the ground state to the first $2^{+}$state and the corresponding excitation energy E2 of even-even nuclei in the recently developed Generalized Differential Equation model. Our findings of large deformations in the exotic neutron-rich regions support the existence of an "Island of Inversion" in the heavy-mass region possibly caused by breaking of the $\mathrm{N}=70$ sub-shell closure.


## 1. Introduction

Studies of the nuclear structure for nuclei lying away from the $\beta$-stable valley of the nuclear chart has been a challenging situation of late, due to identification of "Islands of Inversion" $[1,2,3,4,5,6]$. In this regard values of the physical quantities such as $B(E 2) \uparrow$ and the corresponding excitation energy $E 2$ of even-even nuclei play very decisive role [7, 8]. Over the years host of such experimental data for these two physical quantities have led Raman et al. [7] and recently Pritychenko et al. [8] to undertake the well-known Oak-Ridge Nuclear Data Project for comprehensive analysis of all such data. In the light of the prevailing scenario theoretical study involving our recently developed Generalized Differential Equation (GDE) model [9, 10, 11] for these two physical quantities has been found to be quite successful[12]. As per the GDE model, both these quantities for a given even-even nucleus are expressed in

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terms of their derivatives with respect to the corresponding neutron and proton numbers $N, Z$. Again the same equation can be further exploited to generate two recursion relations for predicting the unknown values.

The present work reiterates the salient features of the model and its crucial findings in predicting exotic deformations leading to identification of possible existence of more "Islands of Inversion".

## 2. The GDE Model for $\boldsymbol{B}(E 2) \uparrow$ and $E 2$

The principal equation of the model[12] valid for both $B(E 2) \uparrow$ and the corresponding excitation energy $E 2$ is given by

$$
\begin{equation*}
\frac{C(N, Z)}{A}=\frac{1}{2}\left[(1+\beta)\left(\frac{\partial C}{\partial N}\right)_{Z}+(1-\beta)\left(\frac{\partial C}{\partial Z}\right)_{N}\right] \tag{1}
\end{equation*}
$$

where $N, Z$ and $A$ refer to the neutron, proton and mass numbers of the given nucleus. $\beta$ is the usual asymmetry parameter ( $N-Z$ )/A of the nucleus. The variable C represents both $B(E 2) \uparrow$ and $E 2$. As we can see, the relation (1) connects both $B(E 2) \uparrow$ and $E 2$ of a given nucleus to their partial derivatives with respect to $N$ and $Z$. The very basis behind its proposition goes to a similar equation being satisfied by the local energy component of the ground-state energy of a nucleus, specifically simulating[13] its shell and deformation behavior in the infinite nuclear matter (INM) model [14, 15, 16, 17] of atomic nuclei. Even though its proposition for these two physical quantities $B(E 2) \uparrow$ and $E 2$ has been made on close analogy with the local energy term of the INM model, it can be treated as a semi-empirical equation as it has been found $[9,11]$ to be satisfied by them by virtue of their slow variation with $N$ and $Z$ locally. Hence the differential Eq. (1) for these two physical quantities may be better termed as a localized semiempirical equation like the difference equations of Ross and Bhaduri[18] and Pattnayak et al.[19]. Then using the usual forward and backward definitions pair wise for both the derivatives we arrive at two recursion relations

$$
\begin{align*}
& C[N, Z]=\frac{N}{A-2} C[N-2, Z]+\frac{Z}{A-2} C[N, Z-2],  \tag{2}\\
& C[N, Z]=\frac{N}{A+2} C[N+2, Z]+\frac{Z}{A+2} C[N, Z+2], \tag{3}
\end{align*}
$$

These relations connecting values of both $B(E 2) \uparrow$ and $E 2$ of the neighboring even-even nuclei from lower to higher mass and vice-verse, are primarily responsible in reaching out from known to the unknown terrain of the nuclear landscape, and thereby facilitate their predictions throughout. Since each of these relations can be rearranged in three different ways one can generate up to six alternate values for a given nucleus. Again each of them being equally probable, the predicted value is then obtained by the arithmetic mean of all those generated values so obtained. The predictions made in the first generation thus obtained for the unknown can be again used along with the known data to generate the next generation predictions and so on.

In the next step, we [12] used these predicted data for calculating the standard deformation parameters such as the quadrupole deformation $\beta_{2}$ and the ratio of $\beta_{2}$ to the Weisskopf single particle $\beta_{2(\mathrm{sp)}}$ for aiding our analysis.

## 3. Results and Discussion

Detailed calculations are carried [12] out following the scheme as laid down above in predicting the desired values of $B(E 2) \uparrow, E 2$ and the deformation parameters for nuclei lying in the exotic regions of the nuclear chart. One should note that the value of the quadrupole deformation $\beta_{2}$ more or less reflects the nature of collectivity of a given nucleus. Its zero value would mean no deformation at all while its finite value would otherwise indicate increasing deformations or collectivity of a given nucleus. In general, its value up to 0.1 more or less reflects spherical nuclei while that of in the range 0.1-0.2 correspond to normal deformations. On the contrary its value in the range 0.3-0.5 reflects strong deformations while its value of $\approx 0.55-0.65$ indicates super deformation. Thus our critical analysis of the resulting data convincingly support possible existence of large collectivity and the consequent exotic deformations for the nuclides ${ }^{30,32} \mathrm{Ne},{ }^{34} \mathrm{Mg},{ }^{60} \mathrm{Ti},{ }^{42,62,64} \mathrm{Cr},{ }^{50,68} \mathrm{Fe},{ }^{52,72} \mathrm{Ni},{ }^{72,70,96} \mathrm{Kr},{ }^{74,76} \mathrm{Sr},{ }^{78,80,106,108} \mathrm{Zr}$, ${ }^{82,84,110,112} \mathrm{Mo},{ }^{140} \mathrm{Te},{ }^{144} \mathrm{Xe},{ }^{147} \mathrm{Ba},{ }^{122} \mathrm{Ce},{ }^{128,156} \mathrm{Nd},{ }^{130,132,158,160} \mathrm{Sm}$ and ${ }^{138,162,164,166} \mathrm{Gd}$.

Our predictions of strong deformation are found to be in close agreement with the experimental observations[3, 6]. In fact such observations lead to the existence of the Islands of Inversion" caused by breaking of the $\mathrm{N}=20$ and $\mathrm{N}=40$ shell-closures by the intruder states respectively from the pf-shell and gd-shell. Thus such agreement with the experimental findings in the medium-low and medium-mass nuclei in the exotic n-rich regions have made us to conjecture the existence of another "island of Inversion" in the heavy-mass region possibly caused by breaking of the $\mathrm{N}=70$ sub-shell closure by the intruder states from the hfp-shell. Thus it appears that the existence of such "Islands of Inversion" in the

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exotic n-rich regions of the nuclear chart may be a general feature of nuclear dynamics waiting to be explored by future experiments.

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